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# Analysis of Occurrence of Digit 2 in Prime Numbers till 1 Trillion

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***Abstract:*** **Prime numbers less than one trillion are probed for occurrence of digit 2 in them. Multiple occurrences of 2's are investigated. The first and last instances of occurrence of all possible repetitions of 2's in them are determined within blocks of higher powers of 10 till 1 trillion.**

***Keywords*: All occurrences, digit 2, prime numbers.**

***Mathematics Subject Classification 2010*: 11Y35, 11Y60, 11Y99.**

## 1. INTRODUCTION

Study of prime numbers has been primarily on two fronts, viz., theoretical level [1] and actual range wise distribution level [4].

This work presents the analysis of occurrence of digit 2 within all primes in ranges of increasing power of 10 till 1 trillion: primes $p$ such that $1 < p < 10^n$, $1 \le n \le 12$. This kind of analysis is done for digits 0 and 1 in [8] and [14], respectively.

## 2. OCCURRENCE OF SINGLE DIGIT 2 IN PRIME NUMBERS

Historic numeral systems compared in [2] seem to use at least 2 symbols. There are many specialties of digit 2. The trend of occurrence of digit 1 in all positive integers is already presented in [11], which is applicable with appropriate changes to digit 2 also. Here we have inspected all prime numbers $p$ in the range $1 < p < 10^{12}$ for occurrence of digit 2.

All, successive, and non-successive occurrences of digits 0 & 1 in all natural numbers are already analyzed in detail in [5], [6], [7] & [11], [12], [13] and those primes till $10^{12}$ in [8], [9], [10] & [14], [15], [16].

Like earlier works, this also required long execution on many computer systems of program written in Java Language and choice of efficient prime generating algorithm [3].

**TABLE 1: NUMBER OF PRIME NUMBERS IN VARIOUS RANGES WITH SINGLE 2 IN THEIR DIGITS**

| Sr. No. | Range | Number of Primes with Single 2 |
|---|---|---|
| 1. | $1 - 10^{1}$ | 1 |
| 2. | $1 - 10^{2}$ | 3 |
| 3. | $1 - 10^{3}$ | 26 |
| 4. | $1 - 10^{4}$ | 314 |
| 5. | $1 - 10^{5}$ | 2,847 |
| 6. | $1 - 10^{6}$ | 25,943 |
| 7. | $1 - 10^{7}$ | 235,673 |
| 8. | $1 - 10^{8}$ | 2,146,695 |
| 9. | $1 - 10^{9}$ | 19,470,523 |
| 10. | $1 - 10^{10}$ | 176,345,441 |
| 11. | $1 - 10^{11}$ | 1,595,422,090 |
| 12. | $1 - 10^{12}$ | 14,421,842,654 |

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## 3. OCCURRENCE OF MULTIPLE DIGITS 2'S IN PRIME NUMBERS

Analysis of all natural numbers containing single, double, triple and multiple number of non-zero digits like 2 in them in ranges of $1 - 10^n$ for $1 \le n \le 12$ is available [11]. Here, number of primes in these ranges containing multiple number of digit 2's have been determined and these are as follows:

**TABLE 2: NUMBER OF PRIME NUMBERS IN VARIOUS RANGES WITH MULTIPLE 2'S IN THEIR DIGITS**

| Sr. No. | Number Range < | Number of Prime Numbers with 2 2's | Number of Prime Numbers with 3 2's | Number of Prime Numbers with 4 2's |
|---|---|---|---|---|
| 1. | $10^3$ | 3 | 0 | 0 |
| 2. | $10^4$ | 37 | 1 | 0 |
| 3. | $10^5$ | 472 | 37 | 1 |
| 4. | $10^6$ | 5,755 | 661 | 34 |
| 5. | $10^7$ | 66,173 | 9,753 | 777 |
| 6. | $10^8$ | 718,179 | 133,493 | 14,757 |
| 7. | $10^9$ | 7,595,056 | 1,692,580 | 235,685 |
| 8. | $10^{10}$ | 78,572,220 | 20,431,380 | 3,411,620 |
| 9. | $10^{11}$ | 799,405,685 | 237,392,844 | 46,259,728 |
| 10. | $10^{12}$ | 8,026,514,388 | 2,680,357,423 | 596,726,180 |

**TABLE 2: Continued …**

| Sr. No. | Number Range < | Number of Prime Numbers with 5 2's | Number of Prime Numbers with 6 2's | Number of Prime Numbers with 7 2's |
|---|---|---|---|---|
| 1. | $10^7$ | 48 | 0 | 0 |
| 2. | $10^8$ | 1,014 | 32 | 1 |
| 3. | $10^9$ | 21,145 | 1,184 | 37 |
| 4. | $10^{10}$ | 380,511 | 28,533 | 1,354 |
| 5. | $10^{11}$ | 6,179,808 | 574,538 | 36,645 |
| 6. | $10^{12}$ | 92,999,191 | 10,357,460 | 822,063 |

**TABLE 2: Continued …**

| Sr. No. | Number Range < | Number of Primes with 8 2's | Number of Primes with 9 2's | Number of Primes with 10 2's | Number of Primes with 11 2's |
|---|---|---|---|---|---|
| 1. | $10^9$ | 1 | 0 | 0 | 0 |
| 2. | $10^{10}$ | 40 | 0 | 0 | 0 |
| 3. | $10^{11}$ | 1,536 | 33 | 1 | 0 |
| 4. | $10^{12}$ | 45,358 | 1,707 | 29 | 0 |

This count of multiple 2's coming as digits in primes in various ranges of $1 - 10^i$ is graphically plotted as follows, where vertical axis in on logarithmic scale.

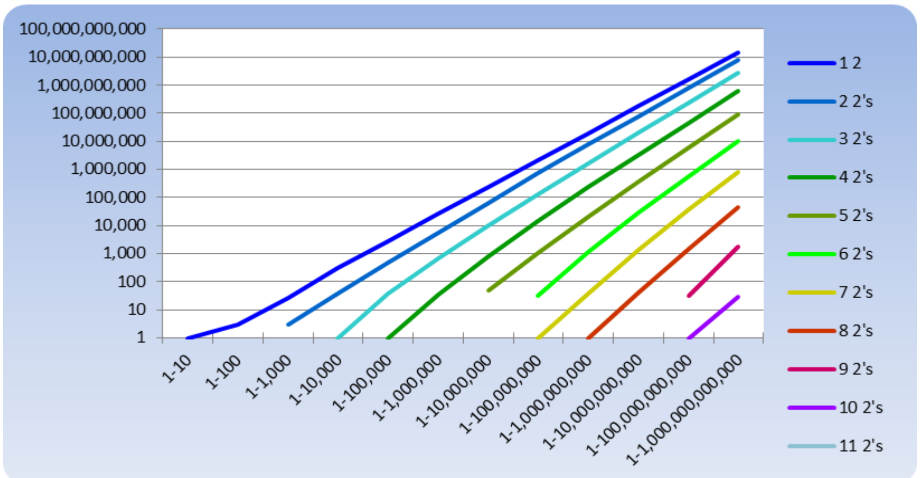


**Figure 1: Number of Primes in Various Ranges with Multiple 2's In Their Digits**

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The percentage of primes containing multiple 2's calculated with respect to number of all such integers with those many 2's in corresponding ranges fluctuates.

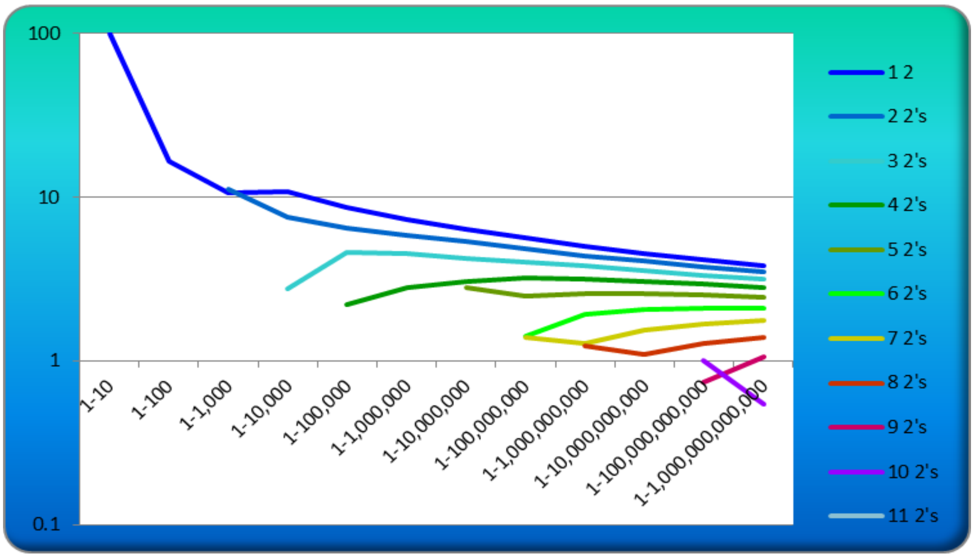


**Figure 2: Percentage of Primes in Various Ranges with Multiple 2's In Their Digits With respect To All Such Integers in Respective Ranges**

The peak observed for 1 2 in the range 1 – 10 is obvious owing to the maximum percentage of 100; the only number 2 with 1 2 in this range is itself a prime. There will be no such peak thereafter as in any range 1 – $10^n$ for occurrences of $n$ 2's, number having $n$ digits 2's would be even and can't be prime.

In prime numbers in all our ranges, comparing with values from [8] and [14], except single occurrence of digits, higher number of digit 2's are more than corresponding number of 0's; while they are less than corresponding number of 1's.

We compare differences of counts of multiple occurrences of digits 2 and 0 in prime numbers in our ranges, splitting them in two groups.

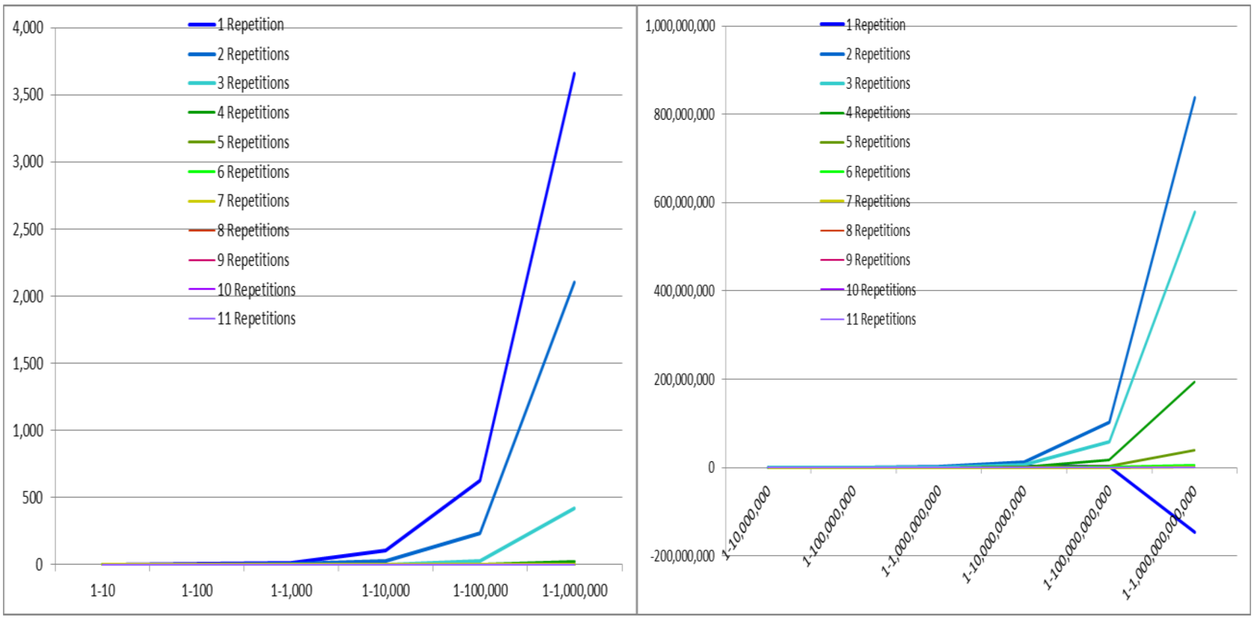


**Figure 3: The Difference of Number of Primes with Multiple 2's Than Those with Multiple 0's**

Now it is turn of comparing difference of digits 2 and 1.

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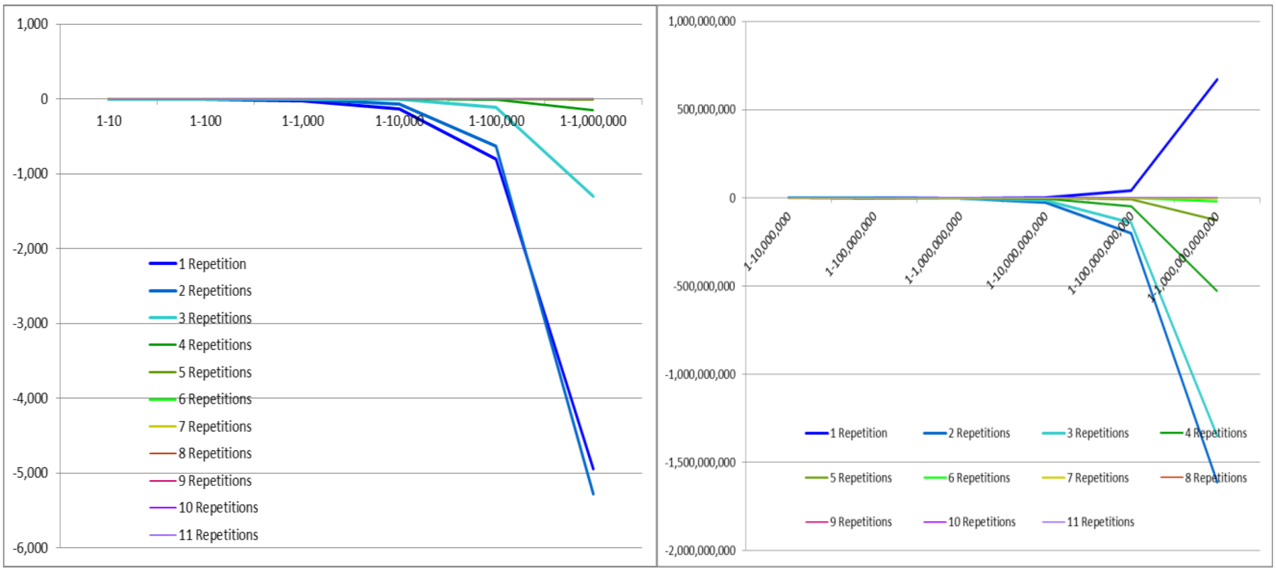


**Figure 4: The Difference of Number of Primes with Multiple 2's Than Those With Multiple 1's**

## 4. FIRST OCCURRENCE OF DIGIT 2 IN PRIME NUMBERS

The first natural number containing single digit 2 is 2 itself! For enough large ranges, for 2 2's, the first example is 22, for 3 it is 222 and so on. It could be formulated simply as

**Formula 1 [11]**: If $n$ and $r$ are natural numbers, then the first occurrence of $r$ number of 2's in numbers in range $1 \le m < 10^n$ is

$$f = \begin{cases} - , & \text{if } r > n \\ \sum_{j=0}^{r-1} \left(2 \times 10^j\right), & \text{if } r \le n \end{cases}$$

Now we are focusing on primes. The first occurrence of $r$ number of 2's in prime numbers in these range $1 \le m < 10^n$ is yet to be fit into a formula. That has made following rigorous determination necessary and precious.

**TABLE 3: FIRST PRIME NUMBERS IN VARIOUS RANGES WITH MULTIPLE 2'S IN THEIR DIGITS**

| Sr. No. | Range | First Prime Number in Range with | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | 1 2 | 2 2's | 3 2's | 4 2's | 5 2's | 6 2's | 7 2's |
| 1. | $1-10^1$ | 2 | - | - | - | - | - | - |
| 2. | $1-10^2$ | 2 | - | - | - | - | - | - |
| 3. | $1-10^3$ | 2 | 223 | - | - | - | - | - |
| 4. | $1-10^4$ | 2 | 223 | 2,221 | - | - | - | - |
| 5. | $1-10^5$ | 2 | 223 | 2,221 | 22,229 | - | - | - |
| 6. | $1-10^6$ | 2 | 223 | 2,221 | 22,229 | - | - | - |
| 7. | $1-10^7$ | 2 | 223 | 2,221 | 22,229 | - | - | - |
| 8. | $1-10^8$ | 2 | 223 | 2,221 | 22,229 | 1,222,229 | - | - |
| 9. | $1-10^9$ | 2 | 223 | 2,221 | 22,229 | 1,222,229 | 20,222,227 | 22,222,223 |
| 10. | $1-10^{10}$ | 2 | 223 | 2,221 | 22,229 | 1,222,229 | 20,222,227 | 22,222,223 |
| 11. | $1-10^{11}$ | 2 | 223 | 2,221 | 22,229 | 1,222,229 | 20,222,227 | 22,222,223 |
| 12. | $1-10^{12}$ | 2 | 223 | 2,221 | 22,229 | 1,222,229 | 20,222,227 | 22,222,223 |

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**TABLE 3: Continued …**

| Sr. No. | Range | First Prime Number in Range with | | | |
|---|---|---|---|---|---|
| | | 8 2's | 9 2's | 10 2's | 11 2's |
| 1. | $1-10^1$ | - | - | - | - |
| 2. | $1-10^2$ | - | - | - | - |
| 3. | $1-10^3$ | - | - | - | - |
| 4. | $1-10^4$ | - | - | - | - |
| 5. | $1-10^5$ | - | - | - | - |
| 6. | $1-10^6$ | - | - | - | - |
| 7. | $1-10^7$ | - | - | - | - |
| 8. | $1-10^8$ | - | - | - | - |
| 9. | $1-10^9$ | 222,222,227 | - | - | - |
| 10. | $1-10^{10}$ | 222,222,227 | - | - | - |
| 11. | $1-10^{11}$ | 222,222,227 | 20,222,222,221 | 22,222,222,223 | - |
| 12. | $1-10^{12}$ | 222,222,227 | 20,222,222,221 | 22,222,222,223 | - |

## 5. LAST OCCURRENCE OF DIGIT 2 IN PRIME NUMBERS

The last instance of $r$ number of 1's in integers in ranges $1-10^n$, $1 \le n \le 12$, has been formulated.

**Formula 2 [11]** : If $n$ and $r$ are natural numbers, then the last occurrence of $r$ number of 2's in numbers in range $1 \le m < 10^n$ is

$$l = \begin{cases} - & \text{, if } r > n \\ \sum_{j=0}^{r-1}\left(2\times 10^j\right) + \begin{cases} 0 & \text{, if } r = n \\ \sum_{j=r}^{n-1}\left(9\times 10^j\right), & \text{if } r < n \end{cases} \end{cases}$$

Recalling that primes don't fit in any such formula, the last prime numbers with $r$ number of 2's in them in ranges $1-10^n$, $1 \le n \le 12$, have been meticulously determined.

**TABLE 4: LAST PRIME NUMBERS IN VARIOUS RANGES WITH MULTIPLE 2'S IN THEIR DIGITS**

| Sr. No. | Number of 2's | Last Prime Number in Range 1 – | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $10^1$ | $10^2$ | $10^3$ | $10^4$ | $10^5$ | $10^6$ | $10^7$ | $10^8$ |
| 1. | 1 | 2 | 29 | 929 | 9,929 | 99,929 | 999,727 | 9,999,929 | 99,999,827 |
| 2. | 2 | - | - | 229 | 9,227 | 99,223 | 999,221 | 9,999,221 | 99,996,229 |
| 3. | 3 | - | - | - | 2,221 | 92,227 | 972,229 | 9,942,223 | 99,982,229 |
| 4. | 4 | - | - | - | - | 22,229 | 922,223 | 9,922,229 | 99,822,227 |
| 5. | 5 | - | - | - | - | - | - | 9,222,229 | 99,222,223 |
| 6. | 6 | - | - | - | - | - | - | - | 72,222,229 |
| 7. | 7 | - | - | - | - | - | - | - | 22,222,223 |
| 8. | 8 | - | - | - | - | - | - | - | - |
| 9. | 9 | - | - | - | - | - | - | - | - |
| 10. | 10 | - | - | - | - | - | - | - | - |
| 11. | 11 | - | - | - | - | - | - | - | - |

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**TABLE 4: Continued …**

| Sr. No. | Number of 2's | Last Prime Number in Range 1 – | | |
|---|---|---|---|---|
| | | $10^9$ | $10^{10}$ | $10^{11}$ |
| 1. | 1 | 999,999,929 | 9,999,999,929 | 99,999,999,829 |
| 2. | 2 | 999,999,229 | 9,999,996,221 | 99,999,999,227 |
| 3. | 3 | 999,992,221 | 9,999,992,227 | 99,999,972,229 |
| 4. | 4 | 999,922,223 | 9,999,822,221 | 99,999,922,229 |
| 5. | 5 | 999,222,221 | 9,999,222,227 | 99,997,222,229 |
| 6. | 6 | 992,222,227 | 9,992,222,221 | 99,992,222,221 |
| 7. | 7 | 822,222,229 | 9,722,222,227 | 99,822,222,229 |
| 8. | 8 | 222,222,227 | 7,222,222,229 | 99,222,222,221 |
| 9. | 9 | - | - | 82,222,222,223 |
| 10. | 10 | - | - | 22,222,222,223 |
| 11. | 11 | - | - | - |

**Remark**: The maximum number of 2's in any prime number in the range 1 – $10^n$ is at most $n - 1$, except $n = 1$.

The integers in various sections of this work form new integer sequences and need separate detail treatment.

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